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Inelastic Buckling of Cylindrical Shells Subjected to Axial Tension and External Pressure

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Introduction

IN the collapse design of oil well casing, the casing is always modeled as having infinite length. Inasmuch as the length/diameter ratio of a typical field joint will vary from 18 to 80, this assumption is reasonable. However, the experimental fixtures from which full-scale collapse data is obtained in steel mills will only accommodate samples having length/diameter ratios in the range of 2-8. The question therefore arises as to by what means experimental results on cylinders of finite length may be translated into design values for longer cylinders. In a recent empirical study of collapse without axial load, Clinedinst[1] has concluded that a length/diameter ratio of 8 is sufficient for the casing to be considered infinite. However, this conclusion is based entirely on 1) analysis of short samples (the largest length/diameter ratio reported was 6:1), and 2) no end constraints. There is currently no method of relating full-scale tests with axial load and clamped end conditions to the behavior of a joint of infinite length. The aim of this analysis is to present specific guidelines by which one can account for the length of the specimen and thereby predict the length/diameter ratio beyond which the effect of the boundary region at the edge of the casing becomes inconsequential.

Analysis

Consider a circular cylindrical shell of length L, mean radius R, and thickness h. The shell is subjected to an axial tensile stress f and an external hydrostatic pressure P on the curved surface measured per unit area of the midsurface. We now introduce the following dimensionless quantities:

$$\lambda=\frac{h}{R},\quad d=\frac{L}{2R},\quad p=\frac{PR}{Eh},\quad \bar{f}=\frac{f}{E},\quad \tau_\epsilon=\frac{\sigma_\epsilon}{E} \tag{1}$$

where σ_e is the effective stress and E the elastic modulus of the material.

Our analysis is based on Sanders' nonlinear shell equations.[2] The rotations about the normal to the midsurface are neglected. The strain components at any point in the shell are related to the membrane strains and curvatures of the midsurface by relations given by Novozhilov.[3] We shall consider only small strains. The material is considered to be elastic-plastic. Both the J_2-incremental theory and the J_2-deformation theory are employed in the formulation. The material constants used in the constitutive relations are derived from the conversion of the relation of the effective stress and the effective strain which is determined from a uniaxial tension test, or an appropriate model of uniaxial stress-strain behavior such as Needleman's curve.[4] The membrane forces and moments are related to the stresses in the shell by integrations through the shell thickness. Since the deformation is symmetric with respect to the midlength, only half of the shell need be considered. All boundary conditions are derived based on Sanders' consistent theory.[2] At the ends of the shell we consider the shell to be clamped.

Prior to buckling the deformation of the shell is axisymmetric. In this case, all physical and geometric quantities are independent of the polar angle. The governing differential equations and boundary conditions for the prebuckling deformation can be derived easily. The calculation of the prebuckling deformation is based on a finite difference scheme with an iterative procedure, i.e., in each incremental step a condition of either loading or unloading is assumed. Two types of loading are considered: proportional loading and constant axial loading. In the latter case, the value of τ_e is checked at each step. When τ_e exceeds a critical value $(\tau_e)_c$, the shell collapses due to exceeding the ultimate strength rather than bifurcation.

In the plastic buckling analysis, the additional physical and geometric quantities introduced by buckling are expressed as sinusoidal functions of the polar angle and the wave number n in the transverse direction. Since our interest lies in finding the critical condition for buckling, second-order terms are neglected. An eigenvalue problem can be formulated. The critical condition for buckling is determined by the characteristic equation of the eigenvalue problem. The value of n corresponding to the lowest load at which buckling occurs will determine the shape of the buckling mode. The details of the analysis are given in Ref. 5.

Numerical Results and Discussion

As pointed out in the introduction, the practical impetus for this study was to obtain a relation between the collapse behavior of short tubes and the collapse behavior of tubes that are essentially infinite in length. However, prior to investigating the effect of length/diameter ratio on collapse behavior, it is instructive to begin with a comparison of the two plasticity theories carried throughout the development. Figure 1 is a plot of predicted dimensionless collapse resistance as a function of axial tension for a cylinder with fixed ends. The collapse predictions are compared to data from Edwards and Miller[6] on small tubes with dimensions $\lambda=0.1227$, $d=16.54$, and a uniaxial stress-strain curve (τ-e curve) that may be fit with Needleman's model

$$\tau/\tau_y=\begin{cases} e/e_y & \text{for } e\le e_y \\ [m(e/e_y)+1-m]^{1/m} & \text{for } e>e_y \end{cases} \tag{2}$$

using $\tau_y=1.679\times10^{-3}$ and $m=5.0$, where τ_y is the dimensionless proportional limit normalized with respect to the modulus of elasticity and e_y is the corresponding strain. Poisson's ratio of the material is considered to be 0.3. The two curves in the figure are labeled by a pair of letters indicating the particular plasticity theory used in analyzing 1) the prebuckled configuration and 2) bifurcation from the fundamental state. For example, the designation I-D indicates that the incremental theory of plasticity (I) was used to determine the prebuckled configuration and the deformation theory of plasticity (D) was used in the buckling analysis.

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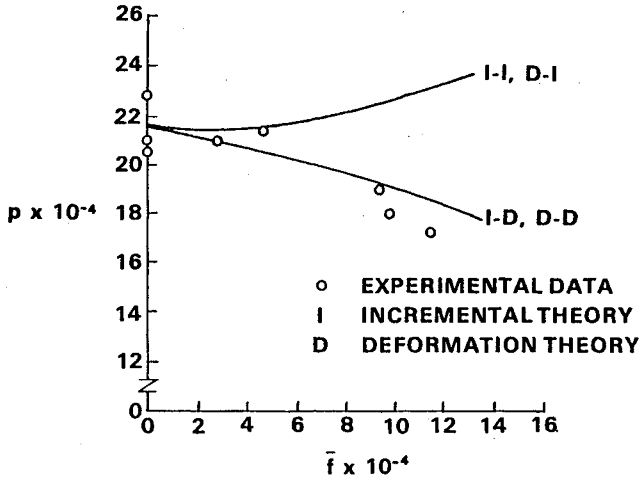

Fig. 1 Comparison of collapse predictions with different plasticity theories (fixed ends).

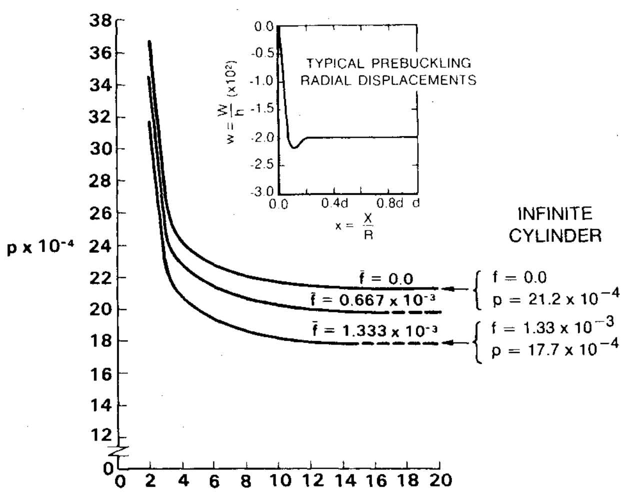

Fig. 2 Effects of length: diameter ratio on plastic collapse (fixed ends). $\lambda = 0.1227$, $\tau_y = 1.679 \times 10^{-3}$, $m = 5$.

The most significant conclusion to be reached from Fig. 1 is that buckling predictions using the incremental theory are unreasonable in that this theory indicates an increase in collapse resistance with increasing axial tension. This anomalous behavior of the incremental theory in predicting plastic buckling loads might have been anticipated in view of earlier results from similar investigations.[7] Notice furthermore from the figure that the choice of plasticity theory for the prebuckling analysis has negligible effect on the predicted collapse load. Finally, it can be seen that the collapse behavior predicted by the deformation theory is capable of reproducing the experimental data with an acceptable degree of accuracy. In view of these conclusions, the deformation theory will be used for the bifurcation analysis throughout the remainder of this discussion.

Returning now to the question of length/diameter ratio, consider Fig. 2 which illustrates typical collapse results for the case of fixed ends and a loading path where $\dot{\bar{f}} \geq 0$, $\dot{p} = 0$ up to a prescribed value of $\bar{f}$, followed by $\dot{p} > 0$, $\dot{\bar{f}} = 0$ to failure. Plotted in the figure is predicted (dimensionless) collapse pressure as a function of length/diameter ratio with and without axial tension. The inset in the figure indicates normalized normal displacement with respect to the shell thickness, w, just prior to buckling for the case $d = 8$, $\bar{f} = 0$, and gives an indication of the extent of the boundary effect for fixed ends. With the exception of very low values of $d(\gtrsim 3)$, and depending on the axial load, all collapses occurred with $n = 2$. The tube material has the same uniaxial stress-strain behavior as that described in the previous paragraph.

Also included in Fig. 2 are arrows indicating the theoretical collapse pressure for an infinite length cylinder calculated from a previous study.[8] Review of this figure indicates that as the length/diameter ratio is increased, the collapse resistance of the tube decreases and approaches a constant value asymptotically. At $d = 8$, the collapse resistance in the absence of axial load is within 4% of this constant value. At $d = 20$, the discrepancy is negligible. Notice further that the presence of axial tension has little effect on the values of these discrepancies.

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